ON THE FIELD ENERGY AND POWER OF WAVEGUIDES AND CAVITIES SYNTHESIZED WITH NONSEPARABLE SOLU - TIONS OF HELMHOLTZ WAVE EQUATION

by

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Abstract

This paper shows that microwave cavities and waveguides synthesized with nonseparable solutions of Helmholtz wave equations have interesting properties concerning field energy and Power. Moreover they have better attenuation and Q factor in comparison with the conventional waveguides and cavities.

Introduction

Recently we studied waveguides and cavities by the use of the nonseparable solution of Helmholtz wave equation. In this paper we consider the field energy and power of these microwave components.

Analysis

There has been shown [1] that nonseparable solutions of Helmholtz wave equation can be used in the synthesis of waveguides and cavities.

Departing from the TM_{11} solution in a rectangular waveguide and a second order combination of wave functions we can write the following solution to the Helmholtz wave equation:

$$\emptyset\,(xy) = \emptyset^{\circ}\,(xy) + C\,\emptyset^{2}\,(xy)$$

where $\emptyset^{\circ}$ (xy), and $\emptyset^{2}$ (xy) are respectively a separable and nonseparable solution and C a perturbation factor.

Depending on the value of C and the dimensions a and b of the rectangular waveguide, various cross-sections can be found different from the original, as seen in fig. 1.

Consider a rectangular waveguide with dimensions $a = \pi$ and b.
$\emptyset$ (xy) is then the longitudinal electric field E_z given by:

$$E_z = \sin x \sin \frac{\pi}{b} y + C\, f(x,y,\pi,b) \qquad (1)$$

with $f(x,y,\pi,b)$ the nonseparable solution of the second order given by:

$$\left(\frac{\pi}{b} x\right)^2 + y^2 \quad \sin x \sin\frac{\pi}{b} y + \frac{\pi}{b} y \sin x \cos \frac{\pi}{b} y$$
$$+ x \cos x \sin \frac{\pi}{b} y + 2x \frac{\pi}{b} \cos x \cos \frac{\pi}{b} y$$

Let's consider the field energy and power. The power in the axial direction of the waveguide is given by:

$$P_z = \frac{1}{2} \frac{\beta^2}{k_c^2} Y_e \int_A E_z E_z^{*}\, dA \qquad (2)$$

where A is the cross-section of the waveguide and Y_e, β respectively the wave admittance and propagation factor.

Substitution of (1) in (2) leads to:

$$P_z = \frac{1}{2} \frac{2}{k_c^2} Y_e \int_A \Big[\sin^2 x \sin^2 \frac{\pi}{b} y$$
$$+ 2\,C\, f(x,y,\pi,b)$$
$$+ C^2 f^2(x,y,\pi,b)\Big]\, dA \qquad (3)$$

In order to compare waveguides with perturbed cross-section with those with rectangular cross-section, we define the relative power:

$$P_{rel} = \frac{P_z}{P'_z} \quad (4)$$

where P_z is the power of the perturbed cross-section and P'_z the power of the rectangular one. After reduction to the same surface fig. 2 shows the relative power flow P_{rel} versus the perturbation factor C with the dimensions a/b as a parameter. At this stage we can conclude that, with respect to energy transport, a waveguide synthesised with nonseparable solutions can support more power depending on the value C and secondly rectangular waveguides need no accurately finished corners. Indeed, some rounded corners give a better energy transport. Investigation of the max. amplitude of the electric field E $|E_z|_{max}$ normalized to the max. amplitude of the field in the rectangular cross-section for constant relative power shown in fig. 3, gives the following features: For constant power, values of $C > 0{,}014$ result in an increasing value of $|E_z|_{max}$.

However high value of C result in significant distortion of the original rectangular cross-section and obviously in a more difficult construction problem of the cavity.

A very good compromise can be found for C between $0{,}014 < C < 0{,}04$.
Which results in $1{,}04 < |E_z|_{max} < 1{,}36$.

Other important properties are the attenuation and the Q factor. Considering the ohmic losses of the walls to be small, perturbation theory can be used. In that case the power flow in the z direction is given by:

$$P(z) = P_o \, e^{-2\alpha z} \quad (5)$$

where α is the attenuation constant.

A straightforward calculation leads us to the following expression for (TM modes)

$$= \frac{\frac{C}{2A}\sqrt{\frac{\varepsilon_o}{\mu_o}}\, R_m}{\sqrt{1 - \frac{\omega_c^2}{\omega^2}}} \cdot \xi_c \quad (6)$$

where C : the circumference of the cross-section

A : the area of the cross-section

R_m: the surface resistance

ξ_c: a dimensionless number given by the relation:

$$\oint_C \frac{1}{\omega_c^2} \left|\frac{E_z}{n}\right|^2 dl = \xi_c\, \varepsilon_o\, \mu_o \frac{C}{A} \int_A |E_z|^2\, dA \quad (7)$$

The relative attenuation

$$\alpha_{rel} = \frac{C/2A\ \xi_c}{C'/2A'\ \xi'_c} \quad (8)$$

compares the attenuation in a perturbed cross-section with non perturbed one.
Fig.4 shows α_{rel} as a decreasing function of the perturbation factor.
This is mainly due to the factor ξ_c.
For cylindrical cavities with perturbed cross-section the Q factor is given by:

$$Q = \frac{\mu}{\mu_c}\,\frac{d}{\delta_c}\,\frac{1}{2\left(1+\xi_c \frac{Cd}{4A}\right)} \quad (9)$$

with d : the length of the cavity

μ_c: the permeability of the metal walls

δ_c : skindepth

In the same manner a relative Q can be defined yielding:

$$Q_{rel} = \frac{1 + \xi_c\ Cd/4A}{1 + \xi'_c\ Cd/4A'} \quad (10)$$

In fig.5 it can be seen that Q_{rel} increases with perturbation.

In conslusion we can say that waveguides and cavities synthesized with nonseparable solutions of the Helmholtz wave equation have better attenuation factor and Q factor in comparison with the conventional rectangular and circular waveguide shapes. Further they have undoubtly interesting properties for micro-wave measurement and power applications.

Alle calculations were done on the I.B.M system 360 of the Rekencentrum of the K.U.L.

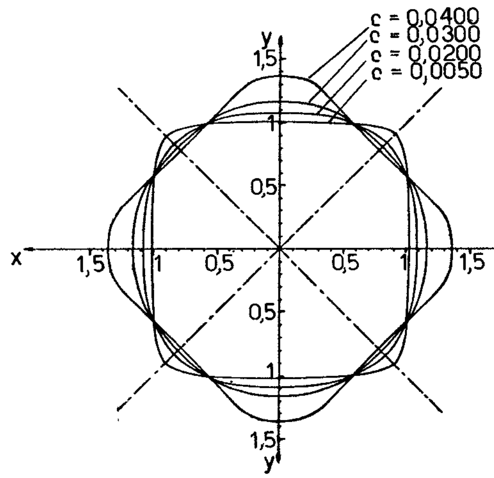
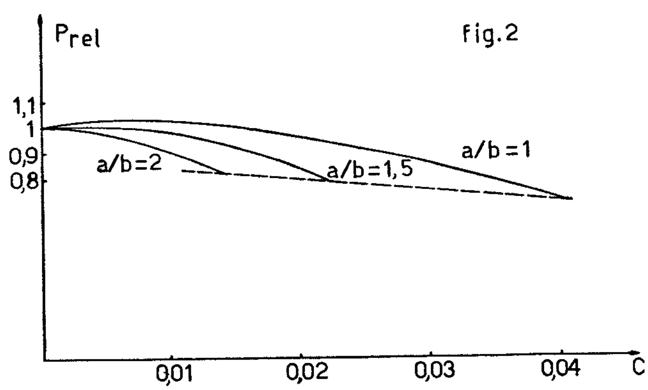
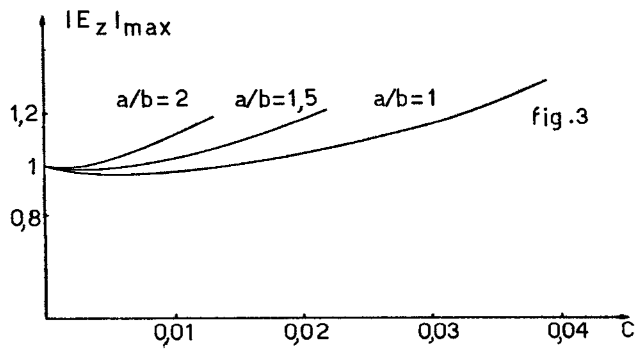


fig. 1

Waveguide cross sections for various values of C.



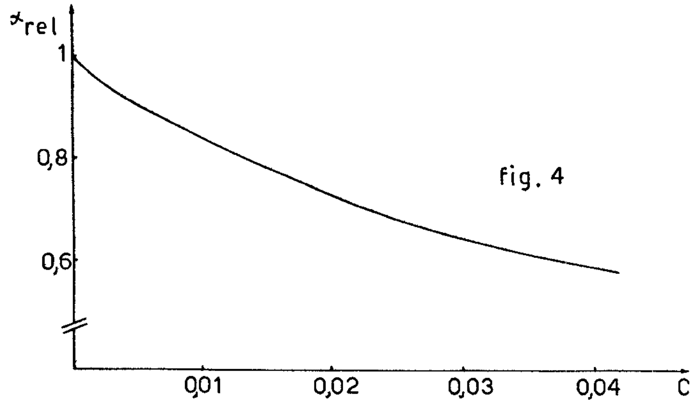
Relative power flow P_{rel} versus the perturbation factor C with a/b as a parameter, and for normalized areas of cross sections and normalized amplitudes.



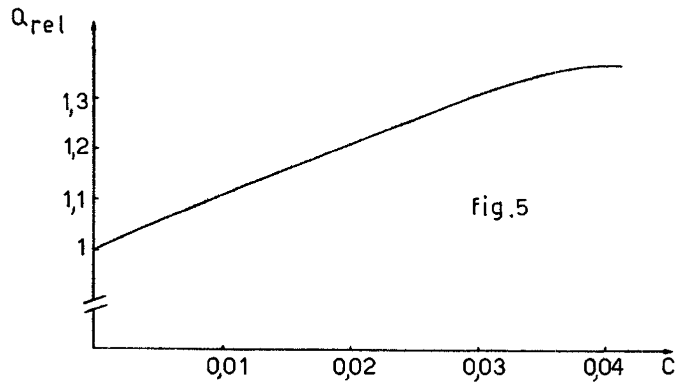
$E_{z\,max}$ versus C with a/b as a parameter.

References:

[1] D.H.Schoonaert, P.J.Luypaert, Electronics Letters 9, 617, (1973)



Relative attenuation factor versus the perturbation factor C.



Relative Q factor versus the perturbation factor C.